Lecture 5 Review

[Q1] In the Examples

- In the examples, some inference rules are missing.

$$\dfrac{[] \vdash \texttt{1} \Rightarrow 1 \qquad \dfrac{[\texttt{x} \mapsto 1] \vdash \texttt{2} \Rightarrow 2 \qquad \dfrac{\begin{array}{c}[\texttt{y} \mapsto 2, \texttt{x} \mapsto 1] \vdash \texttt{x} \Rightarrow 1 \\ [\texttt{y} \mapsto 2, \texttt{x} \mapsto 1] \vdash \texttt{y} \Rightarrow 2\end{array}}{[\texttt{y} \mapsto 2, \texttt{x} \mapsto 1] \vdash \texttt{x + y} \Rightarrow 3}}{[\texttt{x} \mapsto 1] \vdash \texttt{let y = 2 in x + y} \Rightarrow 3}}{[] \vdash \texttt{let x = 1 in let y = 2 in x + y} \Rightarrow 3}$$

[Q1] In the Examples

- In the examples, some inference rules are missing.

$$\dfrac{\dfrac{[] \vdash \mathtt{2} \Rightarrow 2 \quad [\mathtt{y} \mapsto 2] \vdash \mathtt{y + 1} \Rightarrow 3}{[] \vdash \mathtt{let\ y\ =\ 2\ in\ y\ +\ 1} \Rightarrow 3} \quad \dfrac{[\mathtt{x} \mapsto 3] \vdash \mathtt{x} \Rightarrow 3 \quad [\mathtt{x} \mapsto 3] \vdash \mathtt{3} \Rightarrow 3}{[\mathtt{x} \mapsto 3] \vdash \mathtt{x\ +\ 3} \Rightarrow 6}}{[] \vdash \mathtt{let\ x\ =\ (let\ y\ =\ 2\ in\ y\ +\ 1)\ in\ x\ +\ 3} \Rightarrow 6} \quad \ldots$$

[Q2] About the Following Example

```
let x = 7
in let y = 2
   in let  y = let x = x - 1
               in x - y
      in (x-8)-y
```

[Q3] How Can I Prove the Following?

let x = true in x $\notin$ P where

$$
\begin{array}{rcl}
P & \rightarrow & E \\
E & \rightarrow & n \\
& | & x \\
& | & E + E \\
& | & E - E \\
& | & \texttt{iszero}\ E \\
& | & \texttt{if}\ E\ \texttt{then}\ E\ \texttt{else}\ E \\
& | & \texttt{let}\ x = E\ \texttt{in}\ E \\
& | & \texttt{read}
\end{array}
$$

[Q3] How Can I Prove the Following?

The inference rules define a set S of triples (ρ,e,v).

$\boxed{\rho \vdash e \Rightarrow v}$

$$\frac{}{\rho \vdash n \Rightarrow n} \qquad \frac{}{\rho \vdash x \Rightarrow \rho(x)}$$

$$\frac{\rho \vdash E_1 \Rightarrow n_1 \qquad \rho \vdash E_2 \Rightarrow n_2}{\rho \vdash E_1 + E_2 \Rightarrow n_1 + n_2} \qquad \frac{\rho \vdash E_1 \Rightarrow n_1 \qquad \rho \vdash E_2 \Rightarrow n_2}{\rho \vdash E_1 - E_2 \Rightarrow n_1 - n_2}$$

$$\frac{}{\rho \vdash \texttt{read} \Rightarrow n} \qquad \frac{\rho \vdash E \Rightarrow 0}{\rho \vdash \texttt{iszero}\ E \Rightarrow \mathit{true}} \qquad \frac{\rho \vdash E \Rightarrow n}{\rho \vdash \texttt{iszero}\ E \Rightarrow \mathit{false}}\ n \neq 0$$

$$\frac{\rho \vdash E_1 \Rightarrow \mathit{true} \qquad \rho \vdash E_2 \Rightarrow v}{\rho \vdash \texttt{if}\ E_1\ \texttt{then}\ E_2\ \texttt{else}\ E_3 \Rightarrow v} \qquad \frac{\rho \vdash E_1 \Rightarrow \mathit{false} \qquad \rho \vdash E_3 \Rightarrow v}{\rho \vdash \texttt{if}\ E_1\ \texttt{then}\ E_2\ \texttt{else}\ E_3 \Rightarrow v}$$

$$\frac{\rho \vdash E_1 \Rightarrow v_1 \qquad [x \mapsto v_1]\rho \vdash E_2 \Rightarrow v}{\rho \vdash \texttt{let}\ x = E_1\ \texttt{in}\ E_2 \Rightarrow v}$$

then [] ⊢ let x = true in x ∉ S

where

$$\begin{array}{rcl} P & \rightarrow & E \\ E & \rightarrow & n \\ & | & x \\ & | & E + E \\ & | & E - E \\ & | & \texttt{iszero}\ E \\ & | & \texttt{if}\ E\ \texttt{then}\ E\ \texttt{else}\ E \\ & | & \texttt{let}\ x = E\ \texttt{in}\ E \\ & | & \texttt{read} \end{array}$$