

Lecture 8 — Design and Implementation of PLs

(4) States

CSE307: Programming Languages

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2026 Spring

Review: Our Language So Far

Our language has expressions and procedures.

Syntax

$$\begin{array}{l} P \rightarrow E \\ E \rightarrow n \\ \quad | x \\ \quad | E + E \\ \quad | E - E \\ \quad | \text{iszero } E \\ \quad | \text{if } E \text{ then } E \text{ else } E \\ \quad | \text{let } x = E \text{ in } E \\ \quad | \text{read} \\ \quad | \text{letrec } f(x) = E \text{ in } E \\ \quad | \text{proc } x E \\ \quad | E E \end{array}$$

Review: Our Language So Far

Semantics

$$\frac{}{\rho \vdash n \Rightarrow n} \quad \frac{}{\rho \vdash x \Rightarrow \rho(x)} \quad \frac{\rho \vdash E_1 \Rightarrow n_1 \quad \rho \vdash E_2 \Rightarrow n_2}{\rho \vdash E_1 + E_2 \Rightarrow n_1 + n_2}$$

$$\frac{\rho \vdash E \Rightarrow 0}{\rho \vdash \text{iszero } E \Rightarrow \text{true}} \quad \frac{\rho \vdash E \Rightarrow n}{\rho \vdash \text{iszero } E \Rightarrow \text{false}} \quad n \neq 0 \quad \frac{}{\rho \vdash \text{read} \Rightarrow n}$$

$$\frac{\rho \vdash E_1 \Rightarrow \text{true} \quad \rho \vdash E_2 \Rightarrow v}{\rho \vdash \text{if } E_1 \text{ then } E_2 \text{ else } E_3 \Rightarrow v} \quad \frac{\rho \vdash E_1 \Rightarrow \text{false} \quad \rho \vdash E_3 \Rightarrow v}{\rho \vdash \text{if } E_1 \text{ then } E_2 \text{ else } E_3 \Rightarrow v}$$

$$\frac{\rho \vdash E_1 \Rightarrow v_1 \quad [x \mapsto v_1]\rho \vdash E_2 \Rightarrow v}{\rho \vdash \text{let } x = E_1 \text{ in } E_2 \Rightarrow v} \quad \frac{[f \mapsto (f, x, E_1, \rho)]\rho \vdash E_2 \Rightarrow v}{\rho \vdash \text{letrec } f(x) = E_1 \text{ in } E_2 \Rightarrow v}$$

$$\frac{}{\rho \vdash \text{proc } x E \Rightarrow (x, E, \rho)}$$

$$\frac{\rho \vdash E_1 \Rightarrow (x, E, \rho') \quad \rho \vdash E_2 \Rightarrow v \quad [x \mapsto v]\rho' \vdash E \Rightarrow v'}{\rho \vdash E_1 E_2 \Rightarrow v'}$$

$$\frac{\rho \vdash E_1 \Rightarrow (f, x, E, \rho') \quad \rho \vdash E_2 \Rightarrow v \quad [x \mapsto v, f \mapsto (f, x, E, \rho')]\rho' \vdash E \Rightarrow v'}{\rho \vdash E_1 E_2 \Rightarrow v'}$$

This Lecture: Adding States to the Language

- So far, our language only had the values produced by computation.
- But computation also has *effects*: it may change the state of memory.
- We will extend the language to support computational effects:
 - Syntax for creating and using memory locations
 - Semantics for manipulating memory states

Motivating Example

- How can we compute the number of times `f` has been called?

```
let f = proc (x) (x)
in (f (f 1))
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- Does the following program work?

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let counter = 0
in let f = proc (x) (let counter = counter + 1
                    in x)
   in let a = (f (f 1))
      in counter
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Motivating Example

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in let f = proc (x) (let counter = counter + 1
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```

- The binding of `counter` is local. We need global *effects*.
- Effects are implemented by introducing *memory (store)* and *locations (reference)*.

Two Approaches

Programming languages support references explicitly or implicitly.

- Languages with explicit references provide a clear account of allocation, dereference, and mutation of memory cells.
 - e.g., OCaml, F#
- In languages with implicit references, references are built-in. References are not explicitly manipulated.
 - e.g., C and Java.

A Language with Explicit References

$$\begin{aligned} P &\rightarrow E \\ E &\rightarrow n \mid x \\ &\mid E + E \mid E - E \\ &\mid \text{iszero } E \mid \text{if } E \text{ then } E \text{ else } E \\ &\mid \text{let } x = E \text{ in } E \\ &\mid \text{proc } x E \mid E E \\ &\mid \text{ref } E \\ &\mid ! E \\ &\mid E := E \\ &\mid E; E \end{aligned}$$

- $\text{ref } E$ allocates a new location, store the value of E in it, and returns it.
- $! E$ returns the contents of the location that E refers to.
- $E_1 := E_2$ changes the contents of the location (E_1) by the value of E_2 .
- $E_1; E_2$ executes E_1 and then E_2 while accumulating effects.

Example 1

- ```
let counter = ref 0
 in let f = proc (x) (counter := !counter + 1; !counter)
 in let a = (f 0)
 in let b = (f 0)
 in (a - b)
```

## Example 1

---

- `let counter = ref 0`  
  `in let f = proc (x) (counter := !counter + 1; !counter)`  
    `in let a = (f 0)`  
      `in let b = (f 0)`  
        `in (a - b)`
- `let f = let counter = ref 0`  
  `in proc (x) (counter := !counter + 1; !counter)`  
`in let a = (f 0)`  
  `in let b = (f 0)`  
    `in (a - b)`

## Example 1

---

- `let counter = ref 0`  
  `in let f = proc (x) (counter := !counter + 1; !counter)`  
    `in let a = (f 0)`  
      `in let b = (f 0)`  
        `in (a - b)`
- `let f = let counter = ref 0`  
      `in proc (x) (counter := !counter + 1; !counter)`  
`in let a = (f 0)`  
   `in let b = (f 0)`  
      `in (a - b)`
- `let f = proc (x) (let counter = ref 0`  
                  `in (counter := !counter + 1; !counter))`  
`in let a = (f 0)`  
   `in let b = (f 0)`  
      `in (a - b)`

## Example 2

---

We can make chains of references:

```
let x = ref (ref 0)
in (!x := 11; !(!x))
```

# Semantics

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Memory is modeled as a finite map from locations to values:

$$\begin{aligned}Val &= \mathbb{Z} + Bool + Procedure + Loc \\ Procedure &= Var \times \mathbf{E} \times Env \\ \rho \in Env &= Var \rightarrow Val \\ \sigma \in Mem &= Loc \rightarrow Val\end{aligned}$$

Semantics rules additionally describe memory effects:

$$\rho, \sigma \vdash E \Rightarrow v, \sigma'$$

# Semantics

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Existing rules are enriched with memory effects:

$$\frac{\overline{\rho, \sigma \vdash n \Rightarrow n, \sigma} \quad \overline{\rho, \sigma \vdash x \Rightarrow \rho(x), \sigma}}{\rho, \sigma_0 \vdash E_1 \Rightarrow n_1, \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow n_2, \sigma_2} \quad \rho, \sigma_0 \vdash E_1 + E_2 \Rightarrow n_1 + n_2, \sigma_2$$
$$\frac{\rho, \sigma_0 \vdash E \Rightarrow 0, \sigma_1}{\rho, \sigma_0 \vdash \text{iszero } E \Rightarrow \text{true}, \sigma_1} \quad \frac{\rho, \sigma_0 \vdash E \Rightarrow n, \sigma_1}{\rho, \sigma_0 \vdash \text{iszero } E \Rightarrow \text{false}, \sigma_1} \quad n \neq 0$$
$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow \text{true}, \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash \text{if } E_1 \text{ then } E_2 \text{ else } E_3 \Rightarrow v, \sigma_2}$$
$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow \text{false}, \sigma_1 \quad \rho, \sigma_1 \vdash E_3 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash \text{if } E_1 \text{ then } E_2 \text{ else } E_3 \Rightarrow v, \sigma_2}$$
$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow v_1, \sigma_1 \quad [x \mapsto v_1] \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash \text{let } x = E_1 \text{ in } E_2 \Rightarrow v, \sigma_2}$$
$$\overline{\rho, \sigma \vdash \text{proc } x \ E \Rightarrow (x, E, \rho), \sigma}$$
$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow (x, E, \rho'), \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2 \quad [x \mapsto v] \rho', \sigma_2 \vdash E \Rightarrow v', \sigma_3}{\rho, \sigma_0 \vdash E_1 \ E_2 \Rightarrow v', \sigma_3}$$

# Semantics

---

Rules for new constructs:

$$\frac{\rho, \sigma_0 \vdash E \Rightarrow v, \sigma_1}{\rho, \sigma_0 \vdash \text{ref } E \Rightarrow l, [l \mapsto v]\sigma_1} \quad l \notin \text{Dom}(\sigma_1)$$

$$\frac{\rho, \sigma_0 \vdash E \Rightarrow l, \sigma_1}{\rho, \sigma_0 \vdash ! E \Rightarrow \sigma_1(l), \sigma_1}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow l, \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash E_1 := E_2 \Rightarrow v, [l \mapsto v]\sigma_2}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow v_1, \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow v_2, \sigma_2}{\rho, \sigma_0 \vdash E_1; E_2 \Rightarrow v_2, \sigma_2}$$

# Example

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$$\overline{\rho, \sigma_0 \vdash \text{let } x = \text{ref } (\text{ref } 0) \text{ in } (!x := 11; !(!x)) \Rightarrow}$$

## Exercise

---

Extend the language with recursive procedures:

```
 $P \rightarrow E$
 $E \rightarrow n \mid x$
| $E + E \mid E - E$
| iszero $E \mid$ if E then E else E
| let $x = E$ in E
| letrec $f(x) = E$ in E
| proc x $E \mid E E$
| ref E
| ! E
| $E := E$
| $E; E$
```

## Exercise (Continued)

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- Domain:

$$\begin{aligned}Val &= \mathbb{Z} + Bool + Procedure + Loc \\ Procedure &= Var \times \mathbf{E} \times Env \\ \rho \in Env &= Var \rightarrow Val \\ \sigma \in Mem &= Loc \rightarrow Val\end{aligned}$$

- Semantics rules:

$$\frac{}{\rho, \sigma_0 \vdash \text{letrec } f(x) = \mathbf{E}_1 \text{ in } \mathbf{E}_2 \Rightarrow}$$

$$\frac{}{\rho, \sigma_0 \vdash \mathbf{E}_1 \ \mathbf{E}_2 \Rightarrow}$$

# A Language with Implicit References

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$$\begin{aligned} P &\rightarrow E \\ E &\rightarrow n \mid x \\ &\mid E + E \mid E - E \\ &\mid \text{iszero } E \mid \text{if } E \text{ then } E \text{ else } E \\ &\mid \text{let } x = E \text{ in } E \\ &\mid \text{proc } x \ E \mid E \ E \\ &\mid x := E \\ &\mid E; E \end{aligned}$$

- In this design, every variable denotes a reference and is mutable.
- $x := E$  changes the contents of  $x$  by the value of  $E$ .

# Examples

---

Computing the number of times `f` has been called:

- ```
let counter = 0
  in let f = proc (x) (counter := counter + 1; counter)
    in let a = (f 0)
      in let b = (f 0)
        in (a-b)
```

Examples

Computing the number of times `f` has been called:

- ```
let counter = 0
 in let f = proc (x) (counter := counter + 1; counter)
 in let a = (f 0)
 in let b = (f 0)
 in (a-b)
```
- ```
let f = let counter = 0
        in proc (x) (counter := counter + 1; counter)
  in let a = (f 0)
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          in (a-b)
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Examples

Computing the number of times `f` has been called:

- `let counter = 0`
 `in let f = proc (x) (counter := counter + 1; counter)`
 `in let a = (f 0)`
 `in let b = (f 0)`
 `in (a-b)`
- `let f = let counter = 0`
 `in proc (x) (counter := counter + 1; counter)`
`in let a = (f 0)`
 `in let b = (f 0)`
 `in (a-b)`
- `let f = proc (x) (let counter = 0`
 `in (counter := counter + 1; counter))`
`in let a = (f 0)`
 `in let b = (f 0)`
 `in (a-b)`

Exercise

What is the result of the program?

```
let f = proc (x)
          proc (y)
            (x := x + 1; x - y)
in ((f 44) 33)
```

Semantics

References are no longer values and every variable denotes a reference:

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Semantics

$$\frac{}{\rho, \sigma \vdash n \Rightarrow n, \sigma} \quad \frac{}{\rho, \sigma \vdash x \Rightarrow \sigma(\rho(x)), \sigma}$$
$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow n_1, \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow n_2, \sigma_2}{\rho, \sigma_0 \vdash E_1 + E_2 \Rightarrow n_1 + n_2, \sigma_2} \quad \frac{\rho, \sigma_0 \vdash E \Rightarrow 0, \sigma_1}{\rho, \sigma_0 \vdash \text{iszero } E \Rightarrow \text{true}, \sigma_1}$$
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Example

```
let f = let count = 0
        in proc (x) (count := count + 1; count)
in let a = (f 0)
    in let b = (f 0)
        in a - b
```

Exercise

Extend the language with recursive procedures:

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Exercise (Continued)

- Domain:

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Parameter-Passing Variations

- Our current strategy of calling a procedure is *call-by-value*. The formal parameter refers to a new location containing the value of the actual parameter:

$$\frac{\begin{array}{l} \rho, \sigma_0 \vdash E_1 \Rightarrow (x, E, \rho'), \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2 \\ [x \mapsto l]\rho', [l \mapsto v]\sigma_2 \vdash E \Rightarrow v', \sigma_3 \end{array}}{\rho, \sigma_0 \vdash E_1 E_2 \Rightarrow v', \sigma_3} \quad l \notin \text{Dom}(\sigma_2)$$

- The most commonly used form of parameter-passing.

Parameter-Passing Variations

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- The most commonly used form of parameter-passing.
- For example, the assignment to `x` has no effect on the contents of `a`:

```
let p = proc (x) (x := 4)
in let a = 3
    in ((p a); a)
```

Parameter-Passing Variations

- Our current strategy of calling a procedure is *call-by-value*. The formal parameter refers to a new location containing the value of the actual parameter:

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- The most commonly used form of parameter-passing.
- For example, the assignment to `x` has no effect on the contents of `a`:

```
let p = proc (x) (x := 4)
in let a = 3
    in ((p a); a)
```

- Under *call-by-reference*, the assignment changes the value of `a` after the call.

Call-By-Reference Parameter-Passing

The location of the caller's variable is passed, rather than the contents of the variable.

- Extend the syntax:

$$\begin{array}{l} E \rightarrow \vdots \\ | E E \\ | E \langle y \rangle \end{array}$$

- Extend the semantics:

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow (x, E, \rho'), \sigma_1 \quad [x \mapsto \rho(y)]\rho', \sigma_1 \vdash E \Rightarrow v', \sigma_2}{\rho, \sigma_0 \vdash E_1 \langle y \rangle \Rightarrow v', \sigma_2}$$

What is the benefit of call-by-reference compared to call-by-value?

Examples

- `let p = proc (x) (x := 4)`
 `in let a = 3`
 `in ((p <a>); a)`

Examples

- `let p = proc (x) (x := 4)`
 `in let a = 3`
 `in ((p <a>); a)`
- `let f = proc (x) (x := 44)`
 `in let g = proc (y) (f <y>)`
 `in let z = 55`
 `in ((g <z>); z)`

Examples

- `let p = proc (x) (x := 4)`
 `in let a = 3`
 `in ((p <a>); a)`
- `let f = proc (x) (x := 44)`
 `in let g = proc (y) (f <y>)`
 `in let z = 55`
 `in ((g <z>); z)`
- `let swap = proc (x) proc (y)`
 `let temp = x`
 `in (x := y; y := temp)`
`in let a = 33`
 `in let b = 44`
 `in (((swap <a>)); (a-b))`

Variable Aliasing

More than one call-by-reference parameter may refer to the same location:

```
let b = 3
in let p = proc (x) proc (y)
      (x := 4; y)
  in ((p <b>) <b>)
```

- A *variable aliasing* is created: `x` and `y` refer to the same location
- With aliasing, reasoning about program behavior is very difficult, because an assignment to one variable may change the value of another.

Lazy Evaluation

- So far all the parameter-passing strategies are *eager* in that they always evaluate the actual parameter before calling a procedure.
- In eager evaluation, procedure arguments are completely evaluated before passing them to the procedure.
- On the other hand, *lazy evaluation* delays the evaluation of arguments until it is actually needed. If the procedure body never uses the parameter, it will never be evaluated.

- Lazy evaluation potentially avoids non-termination:

```
letrec infinite(x) = (infinite x)
in let f = proc (x) (1)
    in (f (infinite 0))
```

- Lazy evaluation is popular in functional languages, because lazy evaluation makes it difficult to determine the order of evaluation, which is essential to understanding a program with effects.

Summary

Our language is now (somewhat) realistic:

- expressions, procedures, recursion,
- states with explicit/implicit references
- parameter-passing variations