Lecture 1 Review

CSE 307

Top-Down Inductive Definition

Let us define a certain subset $\boldsymbol{S}$ of natural numbers ($\mathbb{N}$) as follows:

Definition ($\boldsymbol{S}$)

A natural number $\boldsymbol{n}$ is in $\boldsymbol{S}$ if and only if

1. $\boldsymbol{n} = 0$, or
2. $\boldsymbol{n} - 3 \in \boldsymbol{S}$.

The definition is *inductive*, because the set is defined in terms of itself. What is the set $\boldsymbol{S}$?

Bottom-up Inductive Definition

An alternative inductive definition of $\boldsymbol{S}$:

Definition ($\boldsymbol{S}$)

$\boldsymbol{S}$ is the *smallest* set such that $\boldsymbol{S} \subseteq \mathbb{N}$ and $\boldsymbol{S}$ satisfies the following two conditions:

1. $0 \in \boldsymbol{S}$, and
2. if $\boldsymbol{n} \in \boldsymbol{S}$, then $\boldsymbol{n} + 3 \in \boldsymbol{S}$.

Rules of Inference

The set $\boldsymbol{S}$ is defined as inference rules as follows:

Definition ($\boldsymbol{S}$)

$$\frac{}{0 \in \boldsymbol{S}} \qquad \frac{\boldsymbol{n} \in \boldsymbol{S}}{(\boldsymbol{n}+3) \in \boldsymbol{S}}$$

Interpret the rules as follows:

"A natural number $\boldsymbol{n}$ is in $\boldsymbol{S}$ iff $\boldsymbol{n} \in \boldsymbol{S}$ can be derived from the axiom by applying the inference rules finitely many times"

Q1: Bottom-up vs Rules of Inference?

- Are they equivalent?

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$\boldsymbol{S}$ is the *smallest* set such that $\boldsymbol{S} \subseteq \mathbb{N}$ and $\boldsymbol{S}$ satisfies the following two conditions:

1. $0 \in \boldsymbol{S}$, and
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vs

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Q2: Does This Lecture Cover Compilers?

Q3: Are There Any Guidelines for the Open Project?

(!) Special Project

Project Goal: Design a programming language that is AI-resistant but human-friendly.

Challenge:

- Existing AI systems (ChatGPT, Claude, etc.) can easily solve programming assignments in conventional languages
- This undermines the learning process and academic integrity

Your Task:

- Design a new programming language where:
  - AI models struggle to write correct solutions
  - Human programmers can still solve assignments effectively
- Consider: syntax design, semantic features, unconventional paradigms
- Deliverable: Language specification + justification of design choices