Lecture 13 — Automatic Type Inference (1)

CSE307: Programming Languages

Minseok Jeon

2026 Spring

The Problem of Automatic Type Inference

Given a program $\boldsymbol{E}$, infer the most general type of $\boldsymbol{E}$ if $\boldsymbol{E}$ can be typed (i.e., $[] \vdash \boldsymbol{E} : \boldsymbol{t}$ for some $\boldsymbol{t} \in \boldsymbol{T}$). If $\boldsymbol{E}$ cannot be typed, say so.

- $\texttt{let}\ f = \texttt{proc}\ (x)\ (x+1)\ \texttt{in}\ (\texttt{proc}\ (x)\ (x\ 1))\ f$
- $\texttt{let}\ f = \texttt{proc}\ (x)\ (x+1)\ \texttt{in}\ (\texttt{proc}\ (x)\ (x\ \mathit{true}))\ f$
- $\texttt{proc}\ (x)\ x$

Automatic Type Inference

- A static analysis algorithm that automatically figures out types of expressions by observing how they are used.
- The algorithm is *sound and complete* with respect to the type system design.
    - (Sound) If the analysis finds a type for an expression, the expression is well-typed with the type according to the type system.
    - (Complete) If an expression has a type according to the type system, the analysis is guaranteed to find the type.
- The algorithm consists of two steps:
    1. Generate type equations from the program text.
    2. Solve the equations.

Generating Type Equations

For every subexpression and variable, introduce type variables and derive equations between the type variables.

Idea: Deriving Equations from Typing Rules

For each expression e and variable x, let t_e and t_x denote the type of the expression and variable. Then, the typing rules dictate the equations that must hold between the type variables.

- $\dfrac{\Gamma \vdash E_1 : \text{int} \qquad \Gamma \vdash E_2 : \text{int}}{\Gamma \vdash E_1 + E_2 : \text{int}}$

$$t_{E_1} = \text{int} \ \wedge \ t_{E_2} = \text{int} \ \wedge \ t_{E_1+E_2} = \text{int}$$

- $\dfrac{\Gamma \vdash E : \text{int}}{\Gamma \vdash \texttt{iszero}\ E : \text{bool}}$

$$t_E = \text{int} \ \wedge \ t_{(\texttt{iszero}\ E)} = \text{bool}$$

- $\dfrac{\Gamma \vdash E_1 : t_1 \to t_2 \qquad \Gamma \vdash E_2 : t_1}{\Gamma \vdash E_1\ E_2 : t_2}$

$$t_{E_1} = t_{E_2} \to \ t_{(E_1\ E_2)}$$

Idea: Deriving Equations from Typing Rules

- $$\frac{\Gamma \vdash E_1 : \mathsf{bool} \qquad \Gamma \vdash E_2 : t \qquad \Gamma \vdash E_3 : t}{\Gamma \vdash \texttt{if } E_1 \texttt{ then } E_2 \texttt{ else } E_3 : t}$$

$$\begin{array}{rcl} t_{E_1} & = & \mathsf{bool} \wedge \\ t_{E_2} & = & t_{(\texttt{if } E_1 \texttt{ then } E_2 \texttt{ else } E_3)} \wedge \\ t_{E_3} & = & t_{(\texttt{if } E_1 \texttt{ then } E_2 \texttt{ else } E_3)} \end{array}$$

- $$\frac{[x \mapsto t_1]\Gamma \vdash E : t_2}{\Gamma \vdash \texttt{proc } x\ E : t_1 \to t_2}$$

$$t_{(\texttt{proc } (x)\ E)} = t_x \to t_E$$

- $$\frac{\Gamma \vdash E_1 : t_1 \qquad [x \mapsto t_1]\Gamma \vdash E_2 : t_2}{\Gamma \vdash \texttt{let } x = E_1 \texttt{ in } E_2 : t_2}$$

$$t_x = t_{E_1} \ \wedge\ t_{E_2} = t_{(\texttt{let } x = E_1 \texttt{ in } E_2)}$$

Example 1

$$\underbrace{\texttt{proc}\ (\underbrace{f}_{t_f})\ \underbrace{\texttt{proc}\ (\underbrace{x}_{t_x})\ \underbrace{(\underbrace{(\underbrace{f}_{t_f}\ \underbrace{3}_{t_5})}_{t_3} - \underbrace{(\underbrace{f}_{t_f}\ \underbrace{x}_{t_x})}_{t_4})}_{t_2}}_{t_1}}_{t_0}$$

Example 2

$$\underbrace{\texttt{proc}\ (\underbrace{\boldsymbol{f}}_{\boldsymbol{t}_{\boldsymbol{f}}})\ (\underbrace{\underbrace{\boldsymbol{f}}_{\boldsymbol{t}_{\boldsymbol{f}}}\ \underbrace{\texttt{11}}_{\boldsymbol{t}_2}}_{\boldsymbol{t}_1})}_{\boldsymbol{t}_0}$$

Example 3

$$\underbrace{\texttt{if}\ \underbrace{x}_{t_x}\ \texttt{then}\ \underbrace{(\underbrace{x}_{t_x} - \underbrace{1}_{t_2})}_{t_1}\ \texttt{else}\ \underbrace{0}_{t_3}}_{t_0}$$

Example 4

$$\underbrace{\texttt{proc}\ (\underbrace{f}_{t_f})\ \underbrace{(\texttt{iszero}\ \underbrace{(\underbrace{f}_{t_f}\ \underbrace{f}_{t_f})}_{t_2})}_{t_1}}_{t_0}$$

Example 5

$$\underbrace{\texttt{proc}\ (\underbrace{x}_{t_x})\ \underbrace{x}_{t_x}}_{t_0}$$

Example 6

$$\underbrace{\texttt{let}\ \underbrace{x}_{t_x} = \underbrace{3}_{t_1}\ \texttt{in}\ \underbrace{(\underbrace{x}_{t_x} + \underbrace{1}_{t_3})}_{t_2}}_{t_0}$$

Example 7

$$\underbrace{\texttt{proc}\ (\underbrace{x}_{t_x})\ \underbrace{\texttt{proc}\ (\underbrace{y}_{t_y})\ \underbrace{(\underbrace{x}_{t_x} + \underbrace{y}_{t_y})}_{t_2}}_{t_1}}_{t_0}$$

Summary

The algorithm for automatic type inference:

1. Generate type equations from the program text.
    - Introduce type variables for each subexpression and variable.
    - Generate equations between type variables according to typing rules.
2. Solve the equations.